

# Mixing in an Intermeshing Twin Screw Extruder Chamber: Combined Cross and Down Channel Flow

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The cross and down channel flows are analyzed in the center of an idealized leakproof intermeshing twin screw extruder chamber. The respective velocity components are assumed to vary only with channel depth. Because the screw flights block the cross channel flow, fluid circulates between two complementary channel depths in the cross channel direction just as predicted for single screw extruders. In addition, fluid circulates between an independent set of channel depths in the down channel direction due to the seal provided by the second screw lands. When the two fluid motions are considered simultaneously, a fluid particle is predicted to follow a complex path over a number of channel depths during its residence time in the extruder. This unique flow also causes particles which are initially near one another to eventually move to significantly distant locations. Furthermore, a wide range of velocities and shear rates is experienced by a fluid particle as it moves to the various channel depths. The strain predicted by two approaches is nearly uniform for the twin screw extruder product in striking contrast to the distribution of absolute strains found in single screw devices. The strain uniformity, wide shear history, and fluid separation predicted by this analysis of a limiting case may help explain the good mixing capabilities of these devices.

## INTRODUCTION

One aspect of mixing in single screw extruders has been characterized in terms of the total amount of strain experienced by a fluid element as it moves through the machine toward the die. To simplify the analysis, strain is usually calculated for the middle part of the channel where the down and cross channel velocities are assumed to be only functions of the channel depth (1-3). Furthermore, the direction of the shear exerted on a fluid element is generally ignored and absolute strain is used to describe mixing. McKelvey (2) states that taking the direction of shear into account would only result in a difference of a numerical factor, while Tadmor and Klein (3) considered that the absolute strain and not the direction is important since fluid "turns over" at the flights.

Considerably less work has been done in describing mixing in twin screw extruders. Wyman (4) has examined the down channel component of velocity for a leakproof twin screw extruder chamber and concluded that the shear rate is the inverted image of a no net flow single screw machine. Further work has revealed that the interaction of the intermeshing second screw lands with the fluid in the chamber produces a uniform abso-

lute strain for down channel flow in a leakproof chamber and could play an important role in down channel mixing (5). Kim, Skatschkow, and Jewmenow (6) predicted strain due to the three components of velocity by averaging the shear rate components over the channel depth far from the intermeshing second screw lands. Their analysis only considers the cross channel circulation between the screw flights but ignores the fluid circulation and resultant strain distribution due to the controlled down channel flow.

Since the intermeshing second screw lands regulate the down channel flow in a twin screw extruder chamber to some degree (depending on leakage), a down channel circulation pattern will also exist, and the combined down and cross channel circulations should be considered. It is the purpose of this paper to examine these down and cross channel circulations in a twin screw extruder. The analysis will treat the fully developed flow in the center part of the chamber as previously done for single screw machines, and the channel is assumed leakproof to simplify the mathematics. Although intermeshing counter-rotating twin screw extruders can approximate the no leakage assumption, real devices will exhibit leakage with the degree influenced by screw characteristics and extruder operating conditions. Co-rotating twin screw extruder chambers are not leakproof, and significant deviations are possible from the

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predictions presented. However, we believe that this analysis is a useful tool in developing a more complete understanding of these complex machines.

### CROSS CHANNEL AND DOWN CHANNEL VELOCITY COMPONENTS

The fluid characteristics are simplified to facilitate the analysis and provide quick insight into the behavior of twin screw extruders. A Newtonian, isothermal fluid of constant density is used in the analysis. The fluid viscosity is very high allowing gravitational and inertial effects to be ignored relative to viscous forces. The flow is assumed steady and laminar.

The twin screw chamber geometry is modified to aid in the analysis as well. First, the perspective of a rotating barrel and stationary screw root is taken, and then the barrel is unwound according to the flat plate approximation (4). Of course, the latter simplification is only strictly reasonable for shallow channels, but by employing it here, the essential features of the flow behavior can be more easily understood. According to this approximation, the barrel moves, at a velocity  $V_b$  ( $\pi D_b N$ ) with components  $V_b \sin \theta$  and  $V_b \cos \theta$  in the cross and down channel directions respectively. The screw root is stationary.

The down channel and cross channel flows for the twin screw chamber are analyzed in much the same manner as for single screw extruders (2,3). Because of the fully developed flow, both velocity components are only functions of the channel depth  $y$ . Therefore, by the continuity equation and the fact that the vertical velocity is zero at the screw root and barrel, the vertical velocity component ( $y$  component) must be zero throughout this middle region.

In the analysis of the cross channel flow, the screw root is stationary ( $V_x = 0$  at  $y = 0$ ) and the barrel is moving ( $V_x = V_b \sin \theta$  at  $y = H$ ). There is assumed to be no fluid leakage past the screw flights, and consequently, the net flow in the cross channel direction is zero. Thus, the cross channel velocity component is simply that for a single screw channel (3):

$$V_x = V_b \sin \theta (y/H)(3y/H - 2) \quad (1)$$

Figure 1(a) illustrates the cross channel velocity profile. Since there is no leakage in the cross channel direction, a fluid particle must travel a closed loop between a plane at a fractional height  $a$ , ( $y/H$ ), and its complementary value  $a^*$  as shown in Fig. 1(a). The relationship between  $a$  and  $a^*$  is the same as for a single screw machine (3):

$$a^* = (1 - a + (1 + 2a - 3a^2)^{1/2})/2 \quad (2)$$

The analysis in the down channel direction is identical to that for the cross channel flow except that the barrel now moves in the down channel direction with a velocity  $V_b \cos \theta$ . Thus, solving the equations of motion yields a down channel velocity distribution given by:

$$V_z = aV_b \cos \theta + \frac{H^2}{2\mu} \left( \frac{\partial p}{\partial z} \right) a(a-1) \quad (3)$$

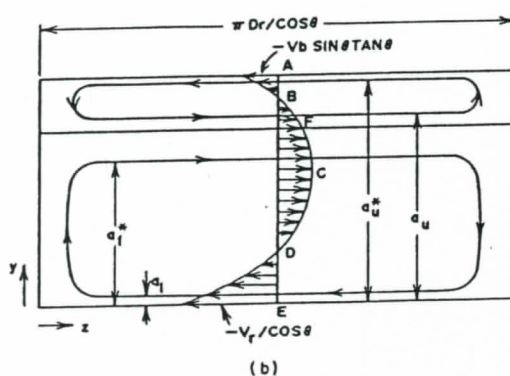
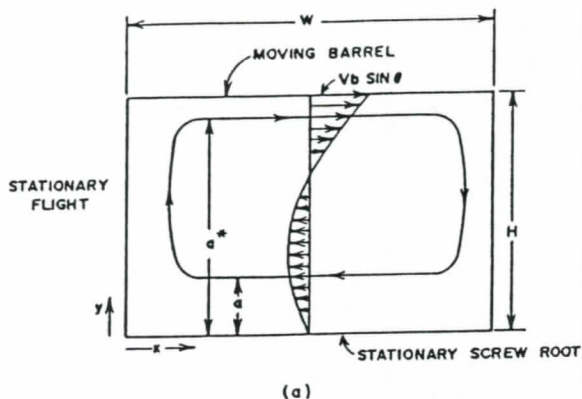


Fig. 1. Velocity profiles in the middle part of the leakproof twin screw chamber: (a) cross channel velocity profile, the screw root and flights being stationary with respect to the  $x$ -direction; (b) down channel velocity profile, the screw root and the barrel moving in the negative  $z$  direction and the second screw lands stationary.

where  $a$  equals  $y/H$  as before. The down channel flow rate is simply the integral of  $V_z$  across the chamber cross section as:

$$Q = \int_0^W \int_0^H V_z dx dy = \frac{WHV_b \cos \theta}{2} - \frac{WH^3}{12\mu} \left( \frac{\partial p}{\partial z} \right) \quad (4)$$

where  $\mu$  is the fluid viscosity.

The extruder chamber is assumed to provide perfect positive displacement action with no down channel leakage past the advancing second screw lands, and the flow rate, therefore, must equal the volumetric displacement rate of the second screw land down the first screw channel. In the actual helical channel, the velocity of advance by the second screw lands is a function of the channel depth; and the average velocity  $\bar{V}_z$  is determined from the integral of the value over the channel height:

$$\bar{V}_z = \frac{\int_0^H \frac{\pi DN}{\cos \theta} dy}{\int_0^H dy} = \frac{(V_b + V_r)}{2 \cos \theta} \quad (5)$$

in which  $D$  is the diameter as a function of channel depth  $y$  and the screw root velocity  $V_r$  equals  $\pi D_r N$ .

The net down channel positive displacement flow rate  $Q$  is  $WH\bar{V}_z$ , the average second screw land velocity times the channel cross sectional area; and equating it to

the fluid flow rate given by Eq. 4, the constant pressure gradient  $\partial p/\partial z$  becomes:

$$\frac{\partial p}{\partial z} = -\frac{6\mu}{H^2}(V_b \sin\theta \tan\theta + V_r/\cos\theta) \quad (6)$$

Combining Eqs 3 and 6, the velocity profile  $V_z$  is:

$$V_z = aV_b \cos\theta + 3a(1-a)(V_b \sin\theta \tan\theta + V_r/\cos\theta) \quad (7)$$

In order to study the down channel circulation pattern, it is advantageous to change the reference frame from a stationary screw root to a stationary second screw land. As a result, the second screw land velocity  $\pi DN/\cos\theta$  must be subtracted from the Eq 7 to give,

$$V_{zs} = a(2-3a)V_b \sin\theta \tan\theta + (1-a)(3a-1)V_r/\cos\theta \quad (8)$$

In Fig. 1(b), the down channel velocity with respect to the stationary screw land,  $V_{zs}$ , is shown.

With respect to the stationary screw lands for the leakproof chamber; the net flow is zero in the down channel direction, and a fluid particle must move between complementary channel depths in the down channel direction in a manner similar to the cross channel direction. However, it can be seen from Fig. 1(b) that unlike the cross channel flow, there are two separate flow patterns for the down channel flow, one with clockwise fluid motion comprising the region from screw root to point F and the other counterclockwise fluid circulation corresponding to the region from the point F to the barrel surface. Thus, as shown in Fig. 1(b), the fractional depths  $a_l$  and  $a^*_l$  (the complementary depth) describe fluid motion in the lower region while  $a_u$  and  $a^*_u$  apply in the upper region. The streamlines calculated previously (5) confirm this analysis. The relationship between any channel depth and its complement due to the down channel circulation can be obtained by integrating the down channel velocity component over the appropriate intervals and equating the flow rates (7) to yield:

$$a^* = \frac{(1+B-a) \pm [(1-B)^2 + 2a(1+B) - 3a^2]^{1/2}}{2} \quad (9)$$

The constant B corresponds to the channel depth which separates the lower circulation region from the upper (point F in Fig. 1(b)) and is given as:

$$B = \frac{1}{1 + \frac{D_b}{D_r} \sin^2\theta} \quad (10)$$

The positive sign is used in Eq 9 when  $a > B$  while a negative sign is employed if  $a < B$ .

#### THE EFFECT OF COMBINED CROSS AND DOWN CHANNEL FLOWS

In the idealized leakproof twin screw extruder, a fluid element will circulate between a pair of complementary channel depths, say  $a_1$  and  $a_2$ , due to the cross channel velocity component just as for a single screw device. However, whereas the fluid element in a single screw extruder will continuously advance toward the die between just this one set of channel depths, in a truly positive displacement twin screw machine, it can only

advance in the down channel direction until it approaches the leading intermeshing second screw land which seals the forward path. Now, if the fluid element is at a depth  $a_1$ , it will move to a complementary channel depth  $a_3$  due to the down channel motion and return toward the trailing second screw land. While traversing backward (with respect to the second screw lands) along the channel length, the fluid element will once again move through a set of complementary depths due to the cross channel motion. Since the complementary channel depths in the down and cross channel directions are generally different, the fluid element will move through a different set of complementary channel depths  $a_3$  and  $a_4$  due to the cross channel motion on its return trip.

When the particle reaches the trailing second screw land, it must again move to a complementary channel depth due to down channel circulation. If the fluid element happens to be at the depth  $a_3$  when it "turns over" at the second screw land, it will move to the original channel depth  $a_1$  (the down channel complement of  $a_3$ ) and move in the forward direction exactly as in the original forward motion. However, if the particle is at the channel depth  $a_4$  when it "turns over", it will move to the down channel complement  $a_5$ . Now on the return trip, the particle may move through the entirely new set of depths  $a_5$  and  $a_6$  due to the cross channel motion. The particle trajectory could be followed indefinitely, and depending upon the location of the fluid element each time it reaches the end of its down or cross channel movement, a wide variety of channel depths may be experienced during the particle's stay in the extruder.

Two particles initially located next to one another can move far apart due to the combined down and cross channel circulations predicted by this simple study. In order to illustrate this phenomenon, consider a typical middle portion of a sealed twin screw extruder chamber as shown in Fig. 2. Two fluid particles, initially separated by a distance of one hundredth of the channel depth and with the lower particle located at a height of one tenth the channel depth above the screw root, are both designated by 0's. Their successive locations after every tenth of the total simulated residence time are shown as pairs of points such as 1 and 1, 2 and 2, and so on. Initially, as long as the particles move to complementary channel depths due to the same velocity component, the shear strain alone is responsible for the growing separation of the particles. However, when one moves to a complementary depth due to the cross channel flow while the other moves to a different complementary depth due to the down channel flow, they will begin to travel independent paths for the rest of their stay in the extruder, and fluid separation is predicted to occur. Thus, as shown in Fig. 2, the particles shown by points 10 and 10 are far apart at the end of the simulated time.

Since a higher residence time means a greater number of cycles for fluid particles and consequently a larger probability of moving to different complementary

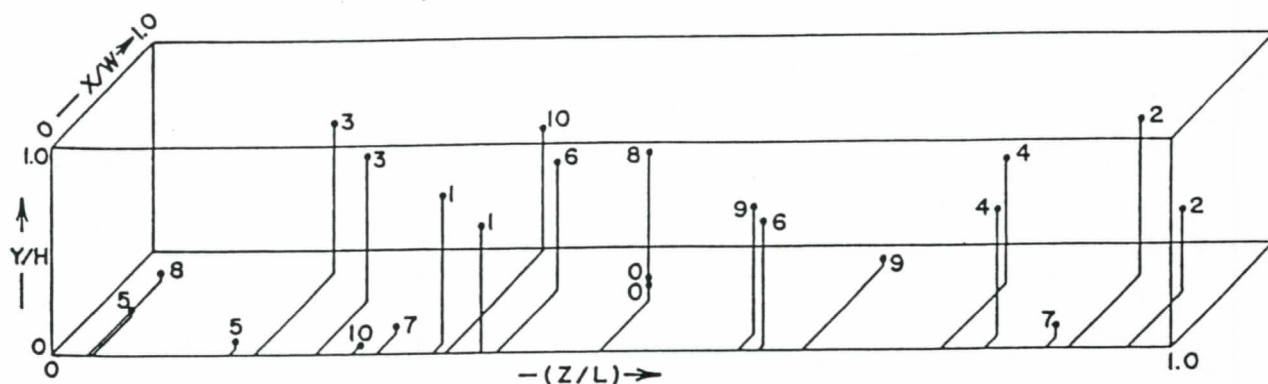


Fig. 2. A representation of the center portion of the twin screw chamber with two fluid particles initially located at points 0 and 0, separated by one hundredth of the channel depth distance, moving through the chamber and finally ending their travel at points 10 and 10. The intermediate positions shown correspond to successive time intervals.

planes, fluid separation is predicted to become more important with increasing residence time in these extruders. Similarly, as the helix angle of a twin screw extruder increases from low values, the cross channel velocity component becomes as effective as the down channel velocity component and fluid separation will become more important. However, the exact nature of the velocities and the predicted fluid separation is not known and deserves further study. Additional attention must be focused on the actual complex velocities near the junction of the screw flights and second screw lands to be certain the predicted separation can in fact occur.

#### THE ABSOLUTE SHEAR STRAIN DISTRIBUTION

In order to accurately analyze mixing in a twin screw extruder, fluid separation and leakage from the chambers should be included along with their effects on the shear strain distribution. It is difficult at this time to predict how much of the total material moving through the twin screw channel will separate due to the combined down as well as cross channel flows since it depends upon factors such as residence time, the helix angle, the ratio of channel height to its width, the influence of the end regions and flights on velocity profiles, and the nature of flow in the corner regions. The effect of leakage is also difficult to accurately include. Therefore, for the present analysis, mixing is described for a leakproof chamber based upon the strain distribution alone across the channel depth taking into account the fluid motion at both sets of complementary channel depths. Actually, better mixing should result than that predicted by the shear strain distribution alone since the fluid separation and leakage from the chamber provide a combining and dividing action for the fluid in addition to the strain.

In the middle part of the twin screw chamber, the  $y$  component of velocity is zero; and a fluid element will move due to only the down and cross channel velocity components. The respective velocity components are assumed constant as long as the fluid particle remains at the same channel depth. Channel end effects are neglected in this approximate analysis. Thus, the shear rate components  $\dot{\gamma}_x$  and  $\dot{\gamma}_z$  in the cross and down channel

directions respectively can be obtained by differentiating the corresponding velocity components with respect to the channel depth  $y$ :

$$\dot{\gamma}_x = \frac{dV_x}{dy} = \frac{V_b}{H} \sin\theta (6a-2) \quad (11)$$

$$\dot{\gamma}_z = \frac{dV_{zs}}{dy} = \frac{V_b}{H} \sin\theta \tan\theta (2-6a) + \frac{V_r}{H} \frac{(4-6a)}{\cos\theta} \quad (12)$$

The resultant shear rate is then obtained by treating the shear rate components  $\dot{\gamma}_x$  and  $\dot{\gamma}_z$  as components of a vector (3):

$$\dot{\gamma} = (\dot{\gamma}_x^2 + \dot{\gamma}_z^2)^{1/2} = \frac{V_r}{H} \left[ \left( \frac{V_b}{V_r} \right)^2 \tan^2\theta (6a-2)^2 + \frac{(4-6a)^2}{\cos^2\theta} + 2 \left( \frac{V_b}{V_r} \right) \tan^2\theta (2-6a)(4-6a) \right]^{1/2} \quad (13)$$

The shear rate given by Eq 13 is assumed to act on the particle until it moves to either the cross channel or the down channel complementary channel depth. The amount of time it will spend at the initial channel depth can be calculated from its initial position ( $x_0, y_0, z_0$ ) and the final position ( $x_f, y_f, z_f$ ) which is determined from the velocity components and the distance across the channel width to the screw flights and along the chamber length to the second screw lands. The time spent at the initial channel depth is:

$$t_0 = \frac{[(x_f - x_0)^2 + (z_f - z_0)^2]^{1/2}}{[V_x^2 + V_{zs}^2]^{1/2}} \quad (14)$$

The shear strain experienced by the particle at the initial channel depth is therefore,

$$\dot{\gamma}_0 = \dot{\gamma}_0 t_0 \quad (15)$$

where  $\dot{\gamma}_0$  is the resultant shear rate in the initial channel depth calculated from Eq 13. This analysis, just as for single screw studies (2,3), does not include the time required for fluid to move between complementary channel depths or consider the possibility that the fluid may "turn over" before reaching the screw flights or second screw lands.

If the particle moves from the initial channel depth due to the cross channel velocity component, the complementary channel depth can be determined from Eq 2. If it moves to the next depth due to the down channel

velocity component, its complementary depth can be determined from Eq 9. The procedure to find the total absolute strain  $\gamma$  from the shear rate  $\dot{\gamma}$  and time  $t$  at the new channel depth is exactly the same as described above, and the calculation is repeated on the computer for all successive complementary depths until the particle leaves the extruder. Adding strains experienced by a fluid particle at each of these channel depths then gives the total absolute shear strain (7).

To calculate the strain for a representative fraction of the fluid in an extruder, strains must be computed for several initial positions as well as various helix angles as a function of total extruder residence time. The total strain distribution shown in Fig. 3 for fluid particles initially located at various channel depths at the center of the  $x-z$  plane varies within about 7 percent of its average value. The shape of the strain distribution curve remains relatively constant as the residence time is increased although the values rise steadily. About a 10 percent deviation in strain distribution from the average was observed over the channel depth for particles initially located at different cross and down channel positions. The strain rapidly increases with helix angle for fluid initially at the middle of the chamber as evident from curve A in Fig. 4, and all other initial locations and residence times show the same general dependence. By contrast, for a single screw extruder, the weighted average strain decreases as the helix angle increases from zero to thirty degrees, remains almost constant between a helix angle of thirty and sixty degrees, and then increases for helix angles greater than sixty degrees(3).

#### ALTERNATE CALCULATION OF SHEAR STRAIN

Some concern arose during the study about using an absolute strain distribution to predict mixing in twin screw extruders. For total absolute strain to be related to laminar mixing, the fluid element must continually be sheared in one direction. If the fluid experiences a

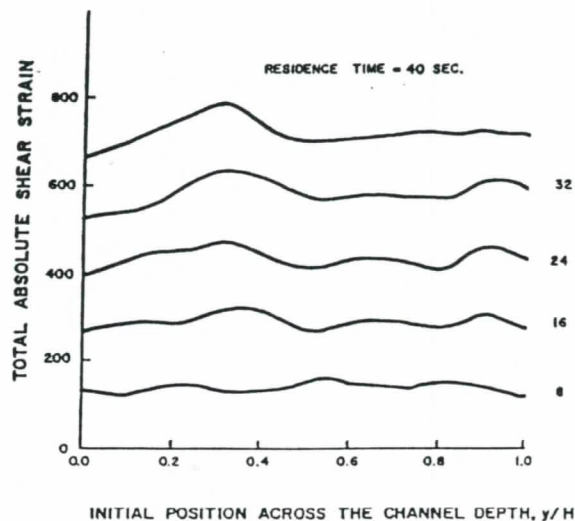


Fig. 3 Total absolute shear strain as a function of initial channel depth and residence time of the extruder, the initial fluid particle position being at the center of the  $xz$  plane. Root diameter = 3 in.; channel depth = 3/4 in.; channel width = 1 in.; screw speed = 1/2 revolution per second; and helix angle = 30°.

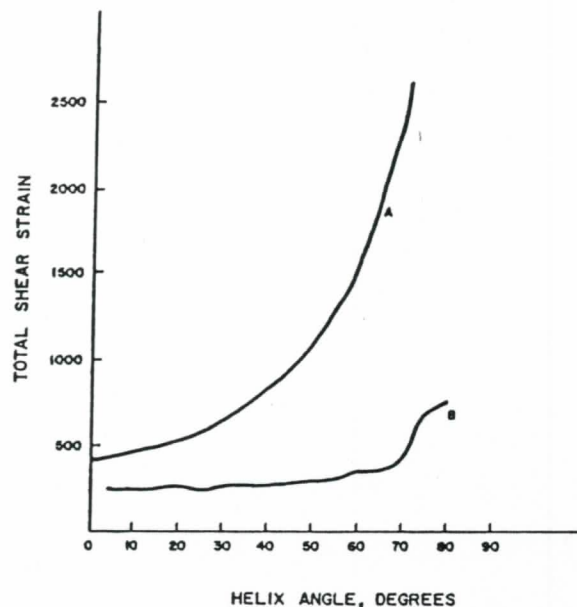


Fig. 4. Total shear strain as a function of helix angle with the initial fluid particle position at the center of the chamber: (A) absolute shear strain; (B) shear strain taking into account shear direction. Root diameter = 3 in.; channel depth = 3/4 in.; channel width = 1 in.; screw speed = 1/2 revolution per second; and residence time = 40 s.

change in the direction of shear, the absolute strain would continue to increase even though the fluid would "unmix" (3). For a single screw extruder, the absolute strain may be useful for describing mixing because the nature of the cross and down channel is such that the strains in much of the region tend to reinforce each other. However, that may not be true for the complex flow case of a twin screw device, and an alternate procedure was employed to calculate mixing based on the shear rate direction.

Equations 11 and 12 were used once again to estimate the shear rate components at any channel depth. When these values are multiplied by the time the fluid remains at the channel depth (Eq 14), the individual shear strain components  $\gamma_x$  and  $\gamma_z$  are obtained.

$$\gamma_x = \dot{\gamma}_x t \quad (16)$$

$$\gamma_z = \dot{\gamma}_z t \quad (17)$$

Running sums  $\Sigma \gamma_x$  and  $\Sigma \gamma_z$  of these strain components are kept, and the total strain at various elapsed times is determined by combining the accumulated strain totals as if they were components of a vector:

$$\gamma = [(\Sigma \gamma_x)^2 + (\Sigma \gamma_z)^2]^{1/2} \quad (18)$$

This procedure is continually repeated over all complementary channel depths exactly as described earlier for the absolute strain calculations.

Figure 5 presents the shear strain distributions predicted by Eq 18. Again, the distribution of strain is somewhat uniform with a deviation of generally less than 12 percent from the average value. As residence time increases, the average level of the strain also increases,

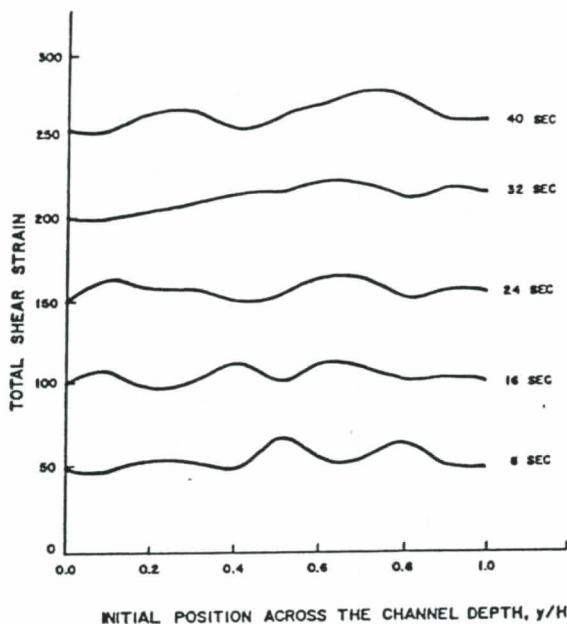


Fig. 5. Total shear strain including shear direction as a function of initial channel depth and extruder residence time. The screw geometry, screw speed, and initial fluid particle position are identical to those described for Fig. 3.

and while the strain distribution pattern varies, the magnitude of the deviation from the average remains essentially the same. Curve B in Fig. 4 shows the influence of the extruder helix angle on the strain, and for the practical range of 5 to 60 degree helix angles, the strain seems to be almost independent of helix angle, in marked contrast to the absolute strain calculation. A similar irregular behavior of shear strain with helix angle above 60 degrees was observed over the channel depth for several positions in the  $x-z$  plane.

#### DISCUSSIONS AND CONCLUSIONS

A number of simplifications were employed in the present analysis. The nature of the flow in the vicinity of the screw flights and the second lands was not taken into account. The strain experienced at the second screw lands is known to be significant (5) while that by the screw flights is commonly ignored for single screw machines. Furthermore, the ideal case of a leakproof twin screw chamber was also employed to facilitate the analysis, although only intermeshing counter-rotating twin screw extruders can approximate this assumption well (8). However, the interesting strain patterns and fluid separation predicted by this simplified analysis would probably be augmented by these other factors if they could be included, and this study may only provide a minimum understanding of some of the mixing capabilities of intermeshing twin screw extruders. It would be very useful to perform a more in depth study of the flow, although a comprehensive analysis would likely be exceedingly difficult. A carefully designed set of experiments might also serve to clarify whether the

predicted mixing capabilities of these devices are possible.

Several interesting observations result from this analysis of an idealized channel. First, due to the predicted interaction of two circulating flows, any element of fluid will move through a variety of channel depths during its stay in the extruder. Consequently, each element will experience the shear rates of each of these channel depths, and a good deal of the extruder product will probably be subjected to shear rates close to the maximum produced by the machine. Exposure to such high shear rates by a large fraction of the material passing through the extruder would aid in tearing apart agglomerates. The nearly uniform shear strain profiles across the channel depth may reflect the exposure to similar shear rate histories. A separation of fluid is also predicted to occur due to the interaction of the two circulating flows perpendicular to each other, but a more detailed analysis is necessary to determine its real nature and significance.

Two approaches for calculating the strain were employed. Although the strain magnitude was higher for the absolute strain method, the strain was predicted to be quite constant over the channel depth by both approaches. In other words, irrespective of the approach used, the total amount of strain is approximately the same for all fluid elements, and as a result, uniform laminar mixing is predicted by this idealized model of a twin screw extruder chamber. However, the absolute strain is predicted to increase sharply with helix angle while the strain that accounts for shear direction predicts a small effect of helix angle.

In comparing strain in a single screw to that in a twin screw extruder, the magnitude of the weighted average strain alone does not appear to be an adequate measure. In a single screw machine, the residence time distribution across the channel depth varies from a low value at a distance of two thirds of the channel depth from the screw root to almost infinity at the screw root and barrel surface (3), and when combined with the shear rate profile, wide variations in the magnitude of strain are evident across the channel depth. In other words, in a single screw extruder, some of the fluid elements are strained considerably while others are not. Consequently, the product from a single screw machine may have a fraction which is poorly mixed (using strain as the measure) while the rest is either adequately mixed or strained beyond the minimum necessary level. On the other hand, in an ideal twin screw extruder with the same weighted average absolute strain, all the material will experience a similar shear-time history and have practically identical strain values. This leads us to believe that the uniformity of the strain distribution is as important as the magnitude of the weighted average value and, therefore, the strain analysis is more meaningful if it also considers the variation of the strain.

#### NOMENCLATURE

- $a$  = fractional channel height,  $y/H$
- $B$  = a constant
- $D$  = diameter, in.

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$H$	= channel depth, in.
$L$	= channel length, in.
$N$	= revolutions per unit time (or revolutions per s)
$p$	= pressure, lb <sub>f</sub> /in. <sup>2</sup>
$Q$	= flow rate, ft <sup>3</sup> /s
$t$	= time, s
$V$	= velocity, in./s
$W$	= channel width, in.
$x, y, z$	= cartesian coordinate distances across the channel width, up the channel depth, and down the channel length respectively.
$\gamma$	= shear strain, dimensionless
$\dot{\gamma}$	= shear rate (or rate of strain), s <sup>-1</sup>
$\theta$	= helix angle, radians or degrees
$\mu$	= fluid viscosity, centipoise

Superscripts

*	= refers to the complementary plane
-	= refers to average quantity

Subscripts

$b$	= refers to the barrel
$f$	= refers to the final position
$l$	= refers to the lower region

$o$	= refers to the initial position
$r$	= refers to the screw root
$u$	= refers to the upper region
$x$	= refers to the cross channel direction
$y$	= refers to the channel depth direction
$z$	= refers to the down channel direction
$zs$	= refers to the down channel velocity with respect to the stationary screw lands

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